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A Model for the Changing Pore Structure and Dust Grain Size Distribution in a Porous Comet Nucleus

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We present a simple model for the flow of dust in a porous icy nucleus of a comet. The model consists of an initial size distribution of pores which are assumed to reside on ''filters'' separated by some characteristic distance, *L*. The dust grains, which have a prescribed initial size distribution, pass through these filters at a given (drag) velocity and block some of the pores. Calculating the probability for pore blocking and particle trapping, we follow the evolution of the pore size distribution and the particle size distribution in both space (as a particular wave of dust passes through successive filters) and time (as successive waves of dust pass through the same filter). © 1997 Academic Press

1. INTRODUCTION

A cometary nucleus consists of water ice with a considerable fraction (1/3 to 1/2) of dust and other volatiles mixed in with the ice. The water ice is believed to be amorphous and to undergo a phase transition to crystalline ice at temperatures above ~100 K. This transition is accompanied by a release of occluded gas (Bar-Nun *et al.* 1987) and is proposed to be responsible for much of the outburst activity observed in comets (see, e.g., Prialnik and Bar-Nun 1992). The phase transition is very sensitive to temperature (Schmidt *et al.* 1989) and hence the behavior of a comet depends on its thermal history. As a result, a great deal of effort has been put into developing thermal evolution models of cometary nuclei.

As the ice sublimates from the nucleus, it leaves behind part of the dust that had been mixed in with the ice. This forms a dust mantle which, in turn, affects the rate of heat and gas flow at the surface of the nucleus. The thickness of this dust mantle is the result of a quasi-steady state between the supply of dust to the surface layer and the removal of that dust by the sublimating gas. Brin and Mendis (1979) made a preliminary study of this effect and determined when such a mantle would be stable. The calculations of Brin and Mendis have since been refined: e.g., Podolak and Herman (1985) and Prialnik and Bar-Nun (1988) include the effect of heat conduction into the nucleus and Rickman *et al.* (1990) take into account trapping of the smaller dust grains by the dust matrix. The rate of formation of this dust mantle clearly depends not only on the rate of gas and dust leaving the surface, but also on the flow of those species through the interior of the nucleus.

The study of gas and dust flow through the porous ice and dust material characteristic of comet nuclei has gained impetus following the close observations of Comet P/Halley, which revealed that its nucleus has a low bulk density, indicating high porosity. Further evidence in favor of porosity is provided by the presence of evolved molecules from volatile ices, which should be depleted in the warm subsurface layers, and thus must come from deeper layers. Pores act as conduits for the transport of gases trapped in comet nuclei (Fanale and Salvail 1987) and dust. Mekler *et al.* (1990) have initiated a detailed study of the effect of porosity on the cometary nucleus and have developed a model of gas flow through a porous medium, allowing for sublimation from the pore walls. This basic model and other similar ones have served to investigate the activity of porous comet nuclei by numerical simulations (e.g., Prialnik and Mekler 1991, Espinasse *et al.* 1991, Steiner and Kömle 1991, Prialnik 1992, Prialnik and Bar-Nun 1992, Espinasse *et al.* 1993, Prialnik *et al.* 1993, Tancredi *et al.* 1994, Benkhoff and Huebner 1995, Seiferlin *et al.* 1995). More recently, attempts have been made to include the flow of dust particles through the pores as well (Orosei *et al.* 1995, Podolak and Prialnik, 1995). Although it is now commonly agreed that porosity must be invoked to properly understand comet behavior, present models are based on very crude assumptions and none of them considers the interaction between the gas and the dust nor the effect of the dust on the porous structure (porosity, pore size distribution). Clearly, the value of the porosity and the pore size distributions, as well as the evolution of those parameters in the course of the comet's life, have important

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effects on the comet's behavior. It is therefore necessary to include such effects in any detailed study of comet evolution.

This paper is the first in a series that will deal with the flow of gas and dust through porous comet nuclei. Its purpose is to present a relatively simple model for the flow of dust through a porous medium (ignoring gravitational forces), taking into account the dust grain size distribution as well as the pore size distribution and their mutual influence due to blocking of small pores by large particles. In Section 2 we outline the model and its inherent assumptions and explain in some detail the method of treating the flow of dust particles. In Section 3 we present the results of calculations. Our conclusions are briefly summarized in Section 4. The application of the semi-analytic model to evolutionary calculations of comet nuclei will be the subject of the next paper in the series.

2. THE MODEL

2.1. General Outline

The following semi-analytic simple model will provide a rough estimate for the evolving dust grain size distribution and pore size distribution inside a comet nucleus, starting from homogeneous initial conditions. For the porous medium we adopt a model of tortuous circular capillaries, allowing the capillary radius to change. Gas is assumed to flow through the pores, dragging with it dust particles. Let L be the typical length over which the pore size (capillary radius) changes. The porous medium may then be described by a series of parallel walls (surfaces) at distance L apart, perforated by circular holes of various radii r. A stream of spherical dust particles of different radii a passes through the walls perpendicularly, at a velocity u determined by the gas flow. A dust grain either crosses a hole or blocks it, the walls acting as filters along the path of the dust grain stream.

The size distribution functions are defined as follows: let the number of dust grains with radii within the interval $(a, a + da)$ per unit volume be given by

$$n_d(a)\, da = \psi_a\, da \tag{1}$$

and the number of holes with radii within the interval $(r, r + dr)$ per unit area be given by

$$n_p(r)\, dr = \eta_r\, dr. \tag{2}$$

The dust grain radii range from a minimum size $a_{\min}$ to a maximum size $a_{\max}$, and similarly, $r_{\min} \le r \le r_{\max}$. The functions ψ_a and η_r change with time t and with distance x from the source of gas and dust. Assuming the walls to be of unit area, the permeability of the medium is given by

$$p(x, t) = \int_{r_{\min}}^{r_{\max}} \pi r^2 \eta_r\, dr, \tag{3}$$

and the dust mass flux (mass of dust crossing a surface per unit time) is given by

$$j(x, t) = \rho_{\mathrm{d}} u \int_{a_{\min}}^{a_{\max}} \frac{4\pi}{3} a^3 \psi_a\, da, \tag{4}$$

where ρ_{d} is the density of a dust particle.

The probability for a dust grain of radius a (flowing with the gas through the pores) to be stopped across a wall (i.e., along a distance L) is given by

$$\xi_a = \int_{r_{\min}}^{a} \eta_r\, dr \Big/ \int_{r_{\min}}^{r_{\max}} \eta_r\, dr. \tag{5}$$

The probability for a hole of radius r to be blocked by a dust particle during a time interval L/u is

$$\varphi_r = \int_{r}^{a_{\max}} \psi_a\, da \Big/ \int_{a_{\min}}^{a_{\max}} \psi_a\, da. \tag{6}$$

Hence the change in the distribution functions ψ_a and η_r over a distance Δx and a corresponding time interval $\Delta t = \Delta x/u$ is given by

$$\psi_a(x + \Delta x, t + \Delta t) = \psi_a(x, t)\left[1 - \xi_a(x, t)\frac{\Delta x}{L}\right] \tag{7}$$

$$\eta_r(x, t + \Delta t) = \eta_r(x, t)\left[1 - \varphi_r(x, t)\frac{\Delta t}{L/u}\right], \tag{8}$$

leading to

$$\frac{d\psi_a}{dt} = -\frac{u\xi_a}{L}\psi_a \tag{9}$$

$$\frac{\partial \eta_r}{\partial t} = -\frac{u\varphi_r}{L}\eta_r. \tag{10}$$

As a crude approximation, justified in the Appendix, we may assume the dust velocity to be equal to the gas velocity, which fulfills the continuity equation

$$up(x, t) = \text{constant} = u_0 p_0, \tag{11}$$

where u_0 is the velocity at the source $x = 0$ and p_0 is the uniform initial permeability. Thus, as pores are being blocked and p decreases, the gas velocity increases in the same proportion.

The porosity $\overline{\Phi}$ of the medium is the fraction of the void space V_{v} within a given volume V. Initially, the void space is the total volume of the pores V_{por} within V; thus $\overline{\Phi}_0 =$

V_{por}/V. As dust particles accumulate between the walls, a bulk dust density builds up within the initially empty pore volume and the fraction of void space decreases. Assuming the solid matrix remains unchanged, we define an effective porosity Φ as the ratio of void space to the pore volume,

$$\Phi \equiv \frac{V_{\text{v}}}{V_{\text{por}}} = \frac{V_{\text{v}}}{V}\frac{V}{V_{\text{por}}} = \frac{\overline{\Phi}}{\overline{\Phi}_0}, \tag{12}$$

the initial value of Φ being unity. The bulk density of the dust ρ within pores is related to the effective porosity by

$$\rho = \rho_{\text{d}}(1 - \Phi), \tag{13}$$

since the gas density is negligible. The change in porosity is given by the mass conservation equation for the dust:

$$-\rho_{\text{d}}\frac{\partial \Phi}{\partial t} + \frac{\partial j}{\partial x} = 0. \tag{14}$$

The change in porosity is linked to the change in permeability by Eqs. (3), (4), and (14), which are coupled through Eqs. (9) and (10).

2.2. Numerical Solution

Equations (7) and (8) may be solved on a discrete grid of surfaces (filters), where distance is measured in multiples of L and time in multiples of L/u. Thus

$$\psi_a(x + L, t + L/u) = \psi_a(x, t)[1 - \xi_a(x, t)] \tag{7$'$}$$

$$\eta_r(x, t + L/u) = \eta_r(x, t)[1 - \varphi_r(x, t)] \tag{8$'$}$$

will provide the grain size distribution in the dust flux, the pore size distribution, and the resulting permeability, as a function of time at any distance from the source. The continuous dust flux at the source is replaced in this case by a series of identical dust waves emitted at constant time intervals L/u_0.

If the filter nearest to the source is labeled 1, the next 2, and so forth, and the dust waves are labeled in chronological order 1, 2, etc., the solution of Eqs. (7′) and (8′) takes the form of two matrices: $\psi(a)_{n,m}$, the size distribution within the nth dust wave after having crossed m filters, and $\eta(r)_{m,n}$, the hole size distribution of the mth filter after n dust waves have passed through it. The global quantities—permeability $p_{m,n}$ and flux $j_{n,m}$—are obtained by integrating over the size distributions, according to Eqs. (3) and (4). The local change in porosity in the space between filters $m - 1$ and m due to the passage of the nth dust wave is given, according to Eq. (14), by

$$\Phi_{m,n} = \Phi_{m,n-1} - (j_{n,m-1} - j_{n,m})/\rho_{\text{d}}\, u. \tag{15}$$

The initial and boundary conditions are of the form $p_{m,0} = p_0$ and $j_{n,0} = j_0$, where p_0 and j_0 are constants, and $\Phi_{m,0} = 1$.

We may assume a power law distribution for the dust grain radii at the source (as indicated by the size distribution of dust grains ejected by Comet P/Halley; McDonnell *et al.* 1986):

$$\psi(x = 0, t) = \psi_{n,0} = Ca^{-\alpha}, \quad a_{\min} \le a \le a_{\max}, \tag{16}$$

or

$$\psi(x = 0, t) = \psi_{n,0} = C'a'^{-\alpha}, \quad \frac{a_{\min}}{a_{\max}} \le a' \equiv \frac{a}{a_{\max}} \le 1.$$

Similarly, we may assume a power law distribution for the initial pore (hole) radii:

$$\eta_r(x, t = 0) = \eta_{m,0} = Dr^{-\beta}, \quad r_{\min} \le r \le r_{\max}. \tag{17}$$

The free parameters of the problem are α, β, $a_{\min}/a_{\max}$, and $r_{\min}/r_{\max}$. In order to reduce their number, we may assume the size distributions of grains and pores to be initially identical. Alternatively, we may adopt normal distributions for the grain and pore radii,

$$\psi_a(x = 0, t) = C\exp[-(a - \bar{a})^2/2\sigma_a^2], \quad 0 < a < \infty, \tag{18}$$

$$\eta_r(x, t = 0) = D\exp[-(r - \bar{r})^2/2\sigma_r^2], \quad 0 < r < \infty, \tag{19}$$

in which case the free parameters would be $\bar{a}, \bar{r}$, σ_a, and σ_r. In either case the constants C and D are determined by the initial (homogeneous) permeability of the medium and the given (constant) dust flux at the source:

$$j_0 = \rho_{\text{d}}\, u_0 \int_{a_{\min}}^{a_{\max}} \frac{4\pi}{3} a^3 \psi(a)_{1,0}\, da, \tag{20}$$

$$p_0 = \int_{r_{\min}}^{r_{\max}} \pi r^2 \eta(r)_{1,0}\, dr. \tag{21}$$

Since we are only interested in relative values, and these are independent of the initial and boundary conditions, we may, without loss of generality, choose $p_0 = 1$ and $j_0/\rho_{\text{d}}\, u_0 = 1$. The resulting distributions and correlations will then depend only on four *physical parameters*, e.g., α, β, $a_{\min}/a_{\max}$, and $r_{\min}/r_{\max}$. We note that the space and time intervals are arbitrary, since L and u_0 are not specified explicitly. Constraints on L and u_0 are imposed by the implied requirement that the porosity is positive everywhere and the flow is nonturbulent; i.e., successive waves do not interfere with each other.

At a given time, i.e., after a number k, say, of dust waves have been emitted at the source, the permeability *profile* throughout the medium will be determined by the series of functions $\eta(r)_{m,k}$, where $m = 1, 2, \ldots$. Similarly, at any fixed place (specified filter i) we can monitor the change in time of the dust flux crossing it, and in particular, the changing grain size distribution, $\psi(a)_{n,i}$, where $n = 1, 2, \ldots$. If consecutive waves are always one filter apart, the dust flux throughout the medium is given by $j_{n,m}$, where $n = 1, 2, \ldots$ and $n + m =$ constant.

When the entire process is over, i.e., after a prescribed number of waves have passed through all the filters, the permeability and the porosity throughout the medium are known and the correlation between them can be obtained. We note that the porous structure of the medium is too complicated to be easily described by a simple geometrical model, and hence the determination of $p(\Phi)$ should be extremely useful, *if* it converges to a unique function (independent of the number of waves and filters assumed).

3. RESULTS

3.1. Power Law Size Distributions

For the purpose of giving a numerical example, we consider a porous medium which is described by 20 filters perforated by randomly distributed holes. We consider up to 50 consecutive waves of dust particles, which cross the series of filters without interfering with each other. The configuration is shown schematically in Fig. 1. The initial size distribution of all the filters is identical and is given by a power law, according to Eq. (17); the size distribution of dust particles in each wave *at the source* is identical and is given by a power law, according to Eq. (16). The size ranges of pores and dust particles are assumed to be identical, namely, $a_{\min} = r_{\min}$ and $a_{\max} = r_{\max}$. Each wave loses a fraction of its particles at each filter crossing; these trapped particles lead to a decrease in the medium's porosity (following Eq. (14)). At the same time, the permeability (total cross section of holes) of each filter decreases to some extent with each wave that has crossed it. Naturally, the filters closest to the source are the ones most affected. Similarly, the first dust waves lose the greatest fraction of their dust particles. If the process were left to run indefinitely, all filters would eventually become completely blocked and impermeable. However, the rate of change of the size distributions, and hence also of the global quatities, slows down considerably. This is illustrated in Fig. 2, where the normalized permeability is plotted as a function of distance from the source of dust (in terms of filter number) at intervals of 10 dust waves, i.e., for increasing times. Near the source the permeability decreases to less than 40% of its initial value, while far away from the source it tends to about 80% of the initial value. Similar results are obtained for various $a_{\max}/a_{\min}$ ratios, the trend being toward lower permeabilities for smaller $a_{\max}/a_{\min}$ ratios.

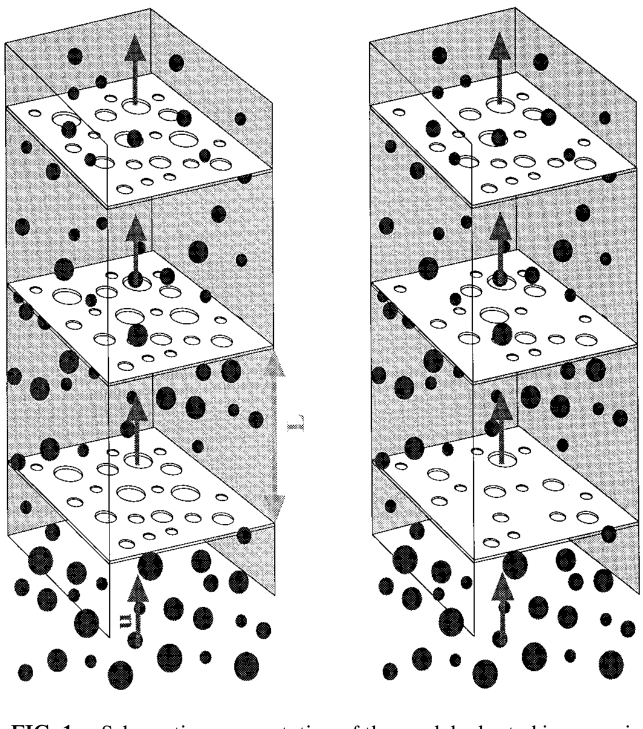

FIG. 1. Schematic representation of the model adopted in numerical calculations. Three filters and two consecutive particle waves passing through them are shown. Note that the second wave encounters holes that have already been blocked by the first wave.

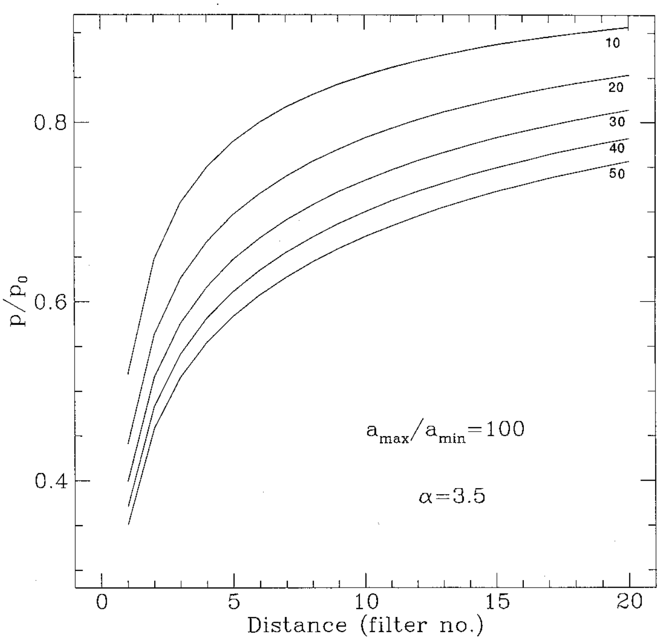

FIG. 2. Normalized permeability as a function of distance from the source in terms of filter number (the first filter is labeled 1) for different waves. The wave numbers (10, 20, 30, 40, 50) are indicated and represent progressive times.

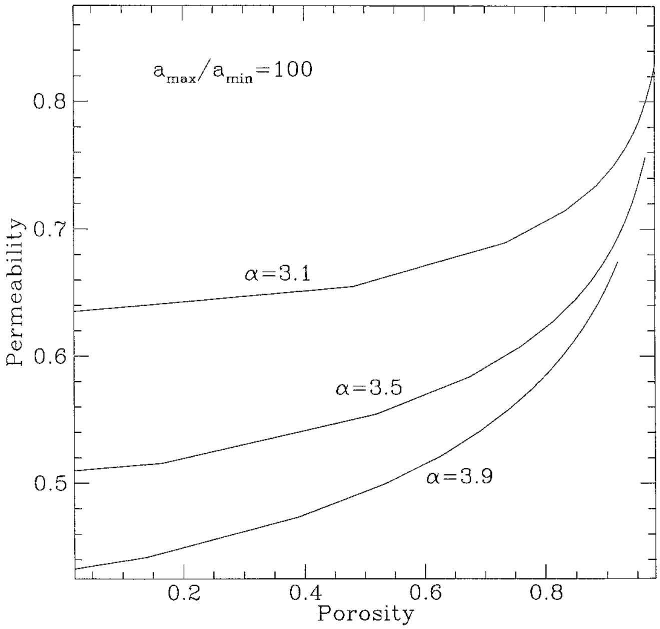

FIG. 3. Normalized permeability p/p_0 as a function of normalized porosity Φ/Φ_0 for different power law distributions of dust grain sizes (the power of the pore size distribution is the same, $\beta = \alpha$).

One of the crucial properties of a porous medium, as regards the flow through it, is the relation between the permeability and the porosity, the former being the factor that determines the rate of flow and the latter the factor that determines the measurable characteristic. Various models are adopted in order to derive this relationship based on assumed geometrical properties of the medium. In the case of a changing porous structure due to trapped dust grains, essentially a stochastic process, the final structure of the medium has no regularity. The dependence of permeability on porosity resulting from our model settles to a unique function (independent of time and distance), determined by the initial size distribution. A few examples are given in Fig. 3, for $a_{max}/a_{min} = 100$ and different values of α (assuming $\alpha = \beta$). For a given porosity, the permeability is lower for a steeper size distribution: the relative number of small pores, which are prone to be blocked, is higher.

An important result of this exercise is the size distribution of the dust grain flux emerging at a large distance from the source, at a time when the system has relaxed and the flux changes imperceptibly with time. The distribution would then depend only on the initial conditions, i.e., the initial size distributions of dust grains and pores. Assuming different power laws for these distributions, but the same ranges of variation ($a_{min} = r_{min}$ and $a_{max} = r_{max}$), we obtain the *average* dust particle size (normalized to its initial value) as a function of α and β, as shown in Fig. 4a. The normalized average pore size far from the source is plotted over the (α, β) plane in Fig. 4b. For a very steep dust grain size distribution (large α), where the overwhelming majority of grains are small, the average radius is barely affected. For a low value of α (relatively numerous large grains), and large β (relatively few large pores), the average grain radius may drop by more than an order of magnitude. The average pore size is higher than the initial pore size, since small pores are blocked with a higher probability than large pores.

3.2. *Normal Size Distributions*

We have repeated the calculations described in the previous section assuming normal initial distributions for both the dust grain radii and the pore radii in order to assess the deviation from this distribution in the emerging dust grain flux. The free parameters in this case are the average radii of grains and pores, $a_{0,av}$ and $r_{0,av}$, and the standard deviations, σ_a and σ_r, respectively. In order to reduce the number of free parameters, these average radii are assumed to be equal. The initial distributions are cut off at $\pm 3\sigma$.

In Fig. 5a we follow the first dust wave as it passes through the first three filters. The size distribution shifts toward smaller radii and the initial symmetry is broken. The second wave is shown in Fig. 5b: the amount of particles that pass through the first filter is reduced as compared to the first wave passing this filter, since some of the holes have been blocked by the first dust wave. However, we note in this case a somewhat slower deviation from the mean of the initial distribution.

Figure 6 is the counterpart of Fig. 4b. The average dust grain size is affected to a much lesser extent than in the case of power law distributions and is not shown. We note that the average pore size is slightly *higher* than the initial pore size.

4. CONCLUSIONS

The flow of gas in a porous comet nucleus is not necessarily confined to a thin outer skin, where most of the surface sublimation takes place, but may involve a layer tens of meters thick below the surface (Prialnik and Bar-Nun 1990). This is due to crystallization of amorphous ice, which (a) releases trapped gases (such as CO or CO_2) and (b) raises the ice temperature sufficiently to cause sublimation from the pore walls. As the ice sublimates, dust grains are loosened into the gas stream. Hence dust grains too may flow through the nucleus at relatively great depths. Since the rate of crystallization is very sensitive to temperature, it usually takes place in a thin layer behind a front, which penetrates the nucleus. Therefore, the source of gas, vapor, and dust grains is almost a point-source at the base of a porous medium, extending from the crystallization front

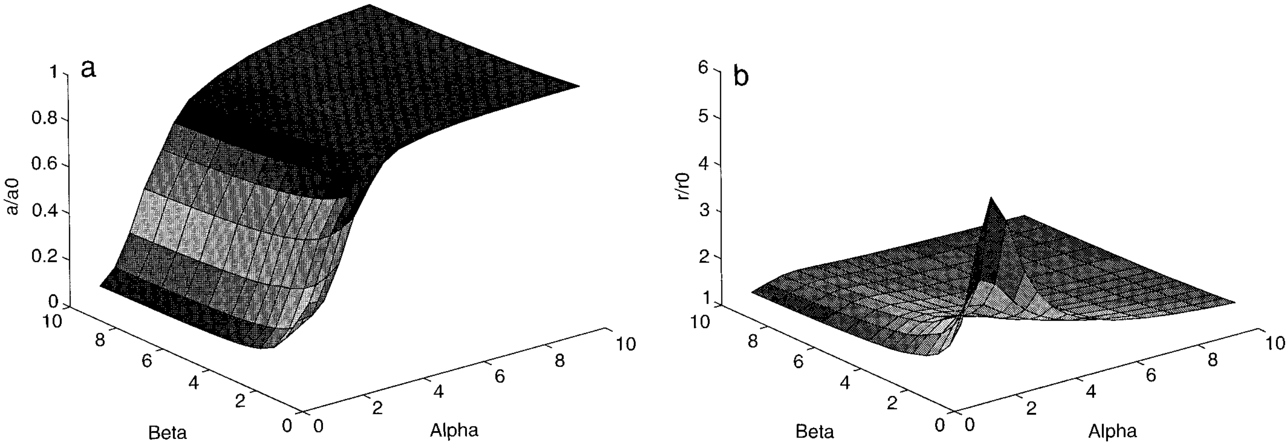

FIG. 4. (a) The average dust grain size in the latest wave of dust particles (50) emerging from the most distant filter (20), normalized to the initial average grain size, and (b) the average pore size for the most distant filter (20) after all particle waves (50) have passed through it, as a function of the powers of the initial distributions of dust radii (α) and of pore radii (β).

to the surface. This is the basic configuration considered in this work. Previous studies have concentrated on the flow of dust near the surface and the formation of a dust mantle by large dust particles as well as by small ones, which become, eventually, trapped between them. In this paper we have considered the possible effect of dust flow and trapping of dust particles through an extended porous medium. This is the first step toward the implementation of these processes in numerical simulations of the behavior of comet nuclei.

We have obtained rough estimates of the changing dust grain size distribution, as dust particles advance through a porous medium. This point is particularly important, in view of the fact that only the dust particles emerging at the surface of a comet are accessible to observation and measurement. In principle, a realistic model of dust flow will help us to deduce the dust distribution in the interior from the distribution in the outcoming flux. When vigorous sublimation takes place at the surface of the nucleus, most of the ejected dust grains are those propelled into the coma by the sublimating gas. At larger heliocentric distances, when the surface activity subsides, the source of dust may be deeper below the surface. Since dust is ejected in distant outbursts, we feel encouraged to consider the flow of dust deeper in the cometary nucleus.

We have also shown how the permeability changes with the porosity of the medium after the passage and trapping of dust. This result may already be used in models of dust mantles of comet nuclei, which are partly permeable to the flow of gas. In Fig. 7a we compare the dependence of permeability on porosity for two widely adopted models—tortuous capillaries and randomly packed spherical grains—with the results of the present model, obtained for $\alpha = \beta = 3.5$. We note more moderate behavior in our case and marked differences at very low and very high porosities. Even more interesting is the comparison between surface to volume ratios, which determine the rate of sublimation (condensation) from (on) the pore walls, shown in Fig. 7b. Here differences between the different models are larger and the present model appears to be the most acceptable.

Finally, this model may serve as a basis for more sophisticated models, which take into account the thermal properties of comet nuclei, their effect on the rate of gas flow, and the interaction between gas and dust.

APPENDIX: THE DUST VELOCITY

Throughout this computation we have assumed that the dust and gas move with the same speed. We now attempt to justify this assumption. The flow of gas through porous comet nuclei is, typically, a free molecular (Knudsen) flow, the mean free path of a gas molecule exceeding the average pore size (e.g., Mekler *et al.* 1990). The drag force on a dust grain of radius a in the Knudsen regime is (Opik 1958)

$$F = \frac{6\pi a^2 \nu}{1.154 l}(u_g - u), \tag{A.1}$$

where ν is the viscosity of the gas, l is the mean free path of a gas molecule, u_g is the gas velocity, and u is the grain velocity. Following Fuchs (1964) we write the viscosity as

$$\nu = \phi \rho_g v_{th} l, \tag{A.2}$$

where ρ_g is the gas density, v_{th} is the thermal velocity of the molecules, and ϕ is a dimensionless constant depending on the way the gas molecules scatter off the dust grain. We take $\phi = 0.35$. The thermal velocity is given by

$$v_{th} = \sqrt{\frac{8}{\pi}\frac{R_g T}{\mu}}, \tag{A.3}$$

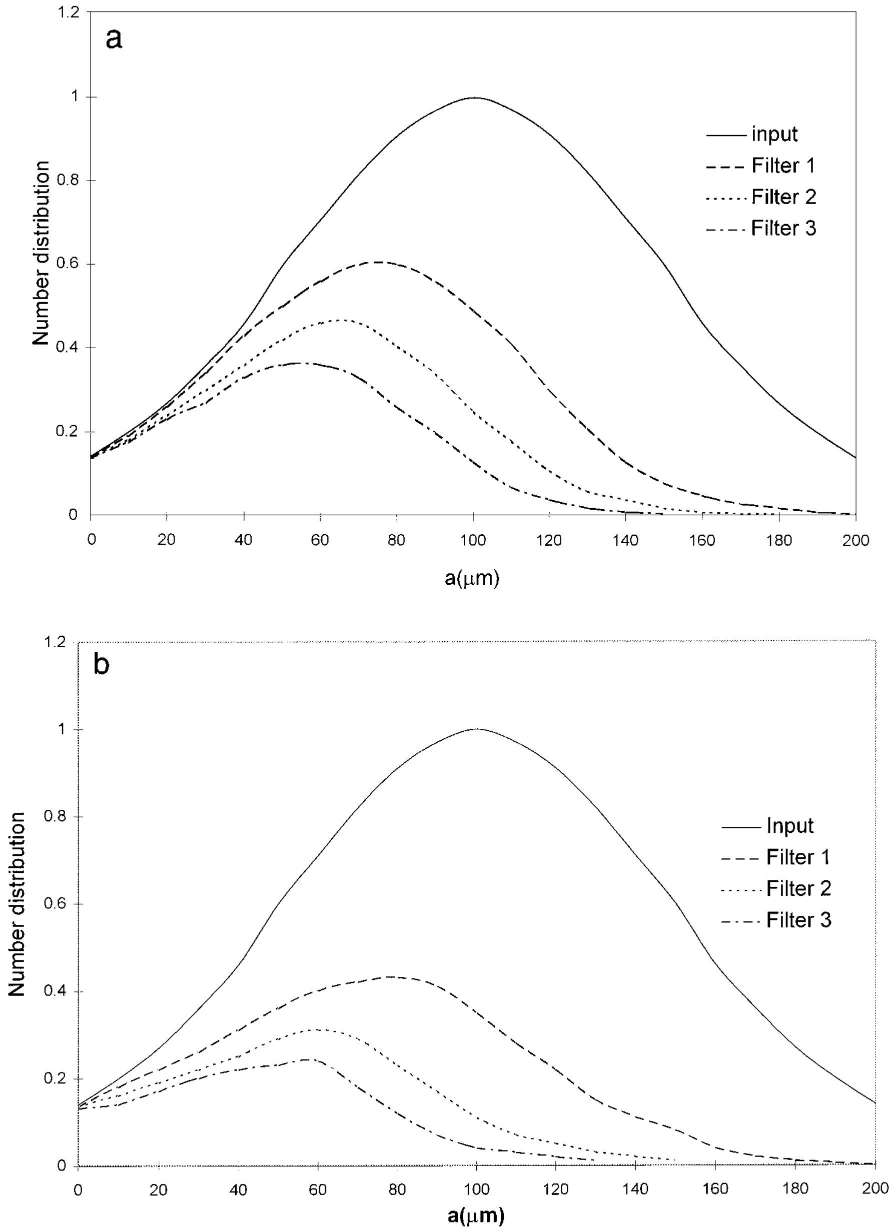

FIG. 5. The grain size distribution within (a) the first dust wave and (b) the second dust wave, as they pass through the first three filters, assuming a normal initial distribution cutoff at 3σ, as shown.

where R_g is the gas constant, T the gas temperature, and μ its mean molecular weight. Substituting these relations in Eq. (A.1), we obtain the drag force in the form

$$F = 5.72a^2\rho_g\sqrt{\frac{8}{\pi}\frac{R_g T}{\mu}}(u_g - u). \tag{A.4}$$

Dividing by the mass of the (spherical) grain and taking the density of the grain to be 2.65 g cm^{-3}, we obtain the grain's acceleration,

$$\frac{du}{dt} = \frac{1}{\tau(a)}(u_g - u), \tag{A.5}$$

where the characteristic time τ, a function of the dust grain radius for given flow conditions, is

$$\tau = 1.33 \times 10^{-4}\frac{1}{\rho_g}\sqrt{\frac{\mu}{T}}a\ s. \tag{A.6}$$

The dust grain velocity (assuming a constant gas velocity) is thus

$$u = u_g(1 - e^{-t/\tau}). \tag{A.7}$$

We now estimate τ for conditions which are typical of cometary interiors

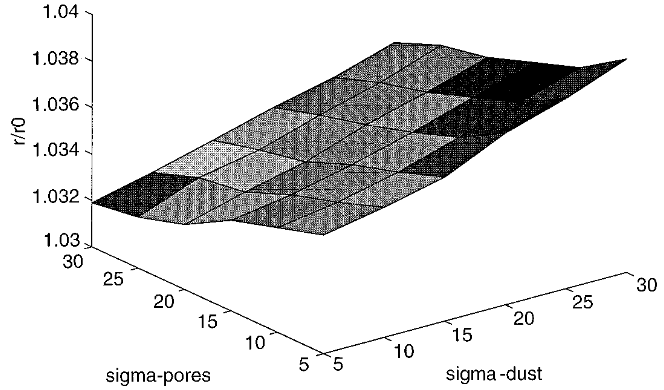

FIG. 6. Same as Fig. 4b, for a normal initial distribution of dust grain radii and pore radii.

(a few meters to a few tens of meters below the surface), where crystallization of amorphous ice takes place and trapped gas (e.g., CO) is released (Prialnik 1992): for $T \simeq 180$ K, $\rho_g \simeq 2.5 \times 10^{-8}$ g cm^{-3}, we find $\tau \approx 0.2$ $(a/1\ \mu\text{m})$, so that even 10-μm particles can reach 90% of the gas velocity in less than 5 sec. For a gas velocity of 0.7 m sec^{-1}, as obtained under such conditions, the particle will have traveled during that time interval a distance of $\sim$2 m, which corresponds to a much smaller radial distance, taking into account that the tortuosity of the ice is expected to be $\sim$4–5 (Whipple and Stefanik 1966). This length scale is considerably smaller than the typical length scale over which conditions change in the interior of the nucleus. Near the surface, where the main driving force is provided by water vapor sublimating from the pore walls, conditions are even more favorable (the gas density being higher).

So far we have neglected the effect of gravity. A gravitational acceleration g would change the solution (A.7) for the velocity to

$$u = (u_g - g\tau)(1 - e^{-t/\tau}), \tag{A.8}$$

assuming the positive direction to be radially outward. Hence the effect is negligible as long as $g\tau \ll u_g$. Since for a constant nucleus density of order 0.5 g cm^{-3} we have $g = 1.4 \times 10^{-7}\ R$ cm sec^{-2}, where R is the radial distance within the comet nucleus (essentially, the cometary radius), the condition for a negligible effect of gravity is

$$Ra \ll 5.4 \times 10^{10}\, u_g \rho_g \sqrt{\frac{T}{\mu}}\ \text{cm}^2. \tag{A.9}$$

For the values adopted above in the evaluation of τ we obtain

$$\left(\frac{R}{1\ \text{km}}\right)\left(\frac{a}{1\ \mu\text{m}}\right) \ll 2 \times 10^4, \tag{A.10}$$

a condition which is amply fulfilled, unless we deal with very large dust grains or very large comets.

In conclusion, we are justified in assuming the dust velocity to be (approximately) equal to the gas velocity.

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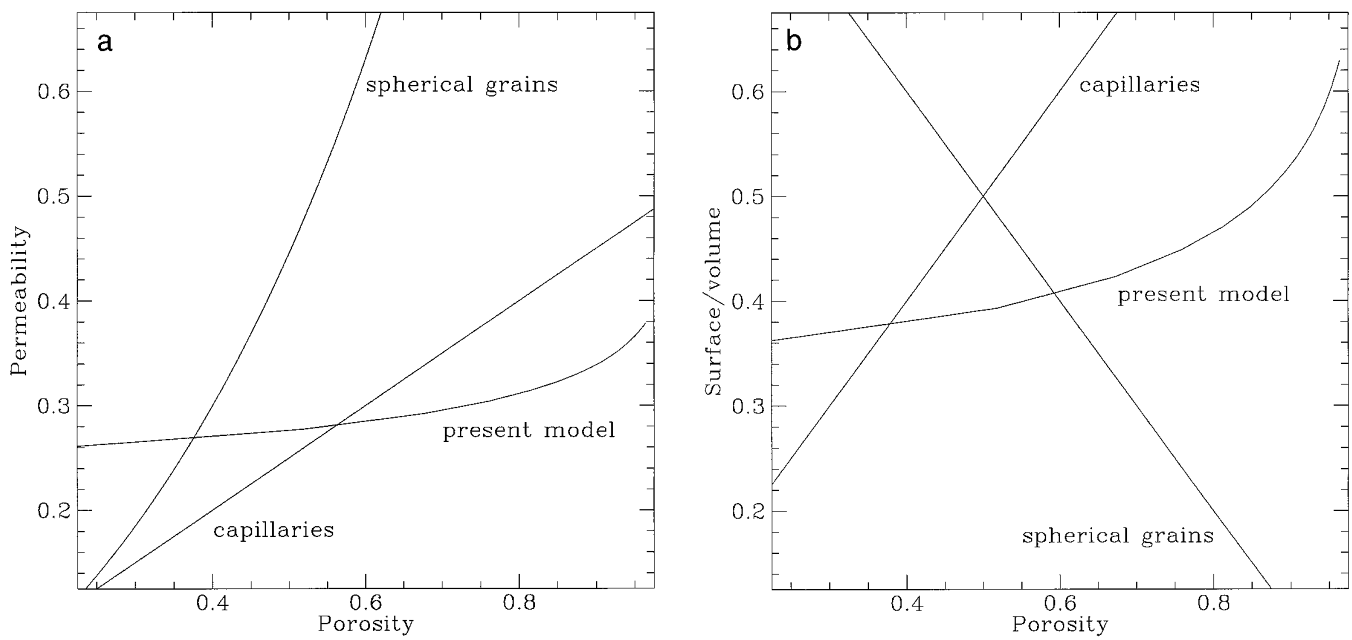

FIG. 7. (a) The dependence of permeability on porosity and (b) the dependence of the surface to volume ratio on porosity, for analytical models of capillaries and of randomly packed spherical grains and for the present model (assuming $\alpha = \beta = 3.5$), as marked.

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